

Indian Statistical Institute, Chennai
MSTAT-2016-17: First Semester

Midterm Examination: Linear Models

Answer all, maximum marks 15.

Duration: 60 minutes

1. Suppose that an experiment with two factors can be modelled by

$$Y_{ijk} = \alpha_i + \beta_{ij} + \varepsilon_{ijk},$$

for $i = 1, 2, \dots, a$, $j = 1, 2, \dots, b_i$ and $k = 1, 2, \dots, n_{ij}$. State the assumptions so that this is a Gauss-Markov model. Represent the above model in the form $Y = X\beta + \epsilon$, when $a = 3$, $b = 2$, and $n_{ij} = 2$. What is the rank of the design matrix X ? [6]

2. Let $E(Y_1) = 2\beta_1 - \beta_2 - \beta_3$, $E(Y_2) = \beta_2 - \beta_4$ and $E(Y_3) = \beta_2 + \beta_3 - 2\beta_4$. Check whether the following parametric functions are estimable? If yes find the BLUE of these parametric functions

a) $\beta_1 + 2\beta_2$

b) $2\beta_2 - \beta_3 - \beta_4$

c) $\beta_2 - \beta_4$ [6]

- 3 Suppose $E(Y_i) = \beta_0 + \beta_1 x_i$, $i = 1, 2, \dots, n$. Construct a test for testing the hypothesis that the slop of the regression line is equal to zero. [5]